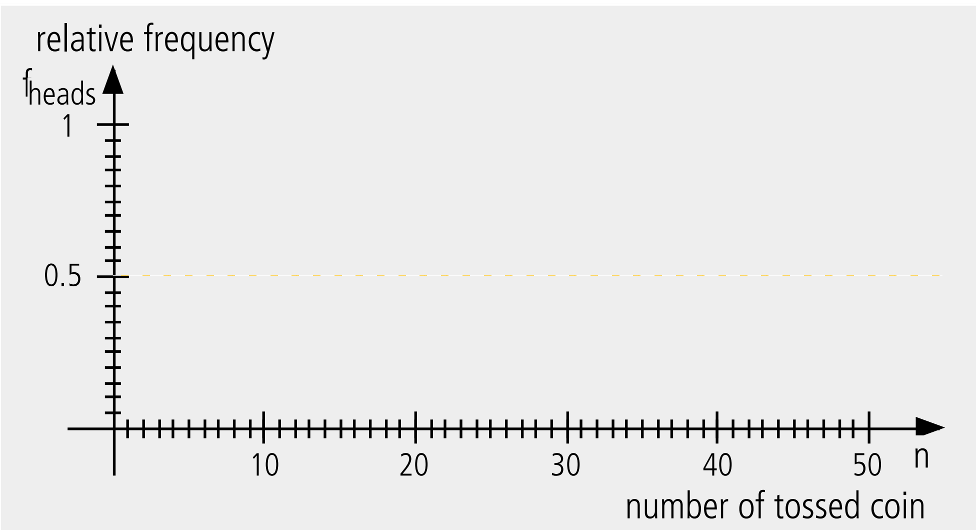
0.5 lessons
1 coin
calculator

LAW OF LARGE NUMBERS

Jakob Bernoulli (1657 - 1705) mathematician
Bernoulli's family was of Belgium origin and had become wealthy in the spice business in Basel. The parents forced Bernoulli to study philosophy and theology but he resented it and took courses in mathematics and astronomy as well.
From 1687 to 1705 he was appointed professor of mathematics at the University of Basel. Leonhard Euler was one of his students and a close friend of the family.
After his death he was first succeeded by his brother Johann and then by his nephew Nikolaus. So for over 100 years the chair of mathematics in Basel belonged to the Bernoulli family.
Jakob Bernoulli's most important contributions to mathematics belong to the field of calculus, mechanics and probability theory. "Ars Conjectandi" (The Art of Guessing) was published in 1713, eight years after his death. With this work he laid the foundation of modern probability theory.

A coin is tossed. The probability that it shows a "head" is 0.5. What does that mean? Is it very likely? Is it very unlikely? Is it absolutely sure? Is it impossible? Will every second tossed coin show "head"?

1 Toss a coin 50 times in a row and count the number of "heads".
Let m = number of "heads" in the first n tosses. After each toss calculate the relative frequency $f_{heads}(n) = \frac{m}{n}$ and plot a dot in a coordinate system as the one below and finally draw a line from one dot to the next one.



The larger the number of tosses the closer to 0.5 the relative frequency seems to stabilize.

This stability is the foundation of probability theory.

The relative frequency tells us something about a specific succession of experiments. If this succession is replaced and idealised by a mathematical model, the relative frequency turns into the probability:

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$$\lim_{n\to\infty}(f) = p$$

So probability is expressed by a number between 0 and 1:

$0 \rightarrow$ impossible $\qquad 1 \rightarrow$ sure

A coin is tossed and the probability to get "heads" is 0.5. This does **not** mean that each "tails" is followed directly by "heads". The outcome of a single experiment can not be predicted.
But it says that if the experiment is repeated often enough the ratio

$$\frac{\text{number of "heads"}}{n}$$

will get closer and closer to 0.5.